

Application of the CLEAN Detector to Low Signal to Noise Ratio Targets

Terry L. Foreman, Ph.D.

Electromagnetic and Sensors System Dept.
Naval Surface Warfare Center Dahlgren Division
Dahlgren, VA, USA

ABSTRACT— This paper expands on the author's previous work by adapting the CLEAN algorithm to address low signal to noise (SNR) targets. The Reformulated CLEAN Detector is presented which is shown to allow the detection of low SNR targets in the presence of large targets. Performance results are presented that show how good performance can be attained by combining the correlator, CLEAN Deconvolver and Reformulated CLEAN Detector

I. INTRODUCTION

The CLEAN algorithm was developed by astronomers [1] to enable them to image dim objects in the presence of bright objects. The problem that CLEAN tries to overcome is that the bright or high amplitude objects have responses that spread out and obscure dimmer objects. CLEAN assumes that the response to a point object is known and is called the point spread function (PSF). It is further assumed that the largest amplitude response in the scene is the location and amplitude of a valid object (as opposed to a spurious response due to noise or the sidelobes of other objects). The largest amplitude and its location is used to position and scale the PSF such that the largest object can be subtracted and thus removed from the scene. This will reveal previously obscured objects. This process is repeated until all that is left in the scene is noise (and the unavoidable errors in the CLEAN algorithm). The previously noted peak amplitudes and their positions are taken as the valid objects in the scene. Thus the CLEAN algorithm is an approach to deconvolution where the PSF is removed from the image. There are a number of papers applying CLEAN to astronomy with [2] providing a recent overview of the application of CLEAN to astronomy. The CLEAN algorithm is also of interest to the radar community. Consequently, there are a number of papers proposing modification to CLEAN and its application to radar problems [3-8].

This paper extends the results of the author's two previous papers on application of the CLEAN Algorithm to the condition of low signal to noise ratio (SNR) targets. The first paper [9] reviewed the CLEAN algorithm, attempted to put it

on a more theoretical ground and derived the CLEAN Deconvolver and the CLEAN Detector. The CLEAN Detector was derived using the Neyman-Pearson Theorem and provides the maximum probability of detection for a specified probability of false alarm. The ambiguity function showed extremely good performance and the mean square error of estimating the target scatterers' amplitudes were shown to be significantly better than the correlator. The second paper [10] addressed the performance of these processors for the condition of Doppler mismatch. It adapted the CLEAN Deconvolver and the CLEAN Detector to unknown Doppler.

For the current work the CLEAN Deconvolver will be taken as the starting point. The CLEAN Deconvolver gets its name from the fact it is the straight forward deconvolution of the received data to recover the scene. This can be seen with the baseband signal model as

$$\mathbf{y} = \tilde{\mathbf{W}}^H \mathbf{c} + \mathbf{n} \quad (1)$$

where multiplication by $\tilde{\mathbf{W}}$ performs the convolution operation and is structured as

$$\tilde{\mathbf{W}} = \begin{bmatrix} 0 & \dots & 0 & \mathbf{w}^t \\ 0 & \dots & \mathbf{w}^t & 0 \\ & \vdots & & \\ \mathbf{w}^t & \dots & 0 & 0 \end{bmatrix},$$

\mathbf{w} is the complex conjugate of the vector of the baseband version of the transmitted signal, \mathbf{c} is the vector of the target complex in reverse range order and \mathbf{n} is Gaussian noise. Note that it is presumed that there is a scatterer in each range cell with unknown complex amplitude.

The CLEAN deconvolver given in [9] is

$$\hat{\mathbf{c}} = (\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H)^{-1} \tilde{\mathbf{W}}\mathbf{y}. \quad (2)$$

Report Documentation Page			Form Approved OMB No. 0704-0188		
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE MAY 2010		2. REPORT TYPE		3. DATES COVERED 00-00-2010 to 00-00-2010	
4. TITLE AND SUBTITLE Application of the CLEAN Detector to Low Signal to Noise Ratio Targets			5a. CONTRACT NUMBER		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S)			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Surface Warfare Center Dahlgren Division, Electromagnetic and Sensors System Dept, Dahlgren, VA, 22448			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)			10. SPONSOR/MONITOR'S ACRONYM(S)		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S)		
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited					
13. SUPPLEMENTARY NOTES See also ADM002322. Presented at the 2010 IEEE International Radar Conference (9th) Held in Arlington, Virginia on 10-14 May 2010.					
14. ABSTRACT This paper expands on the author's previous work by adapting the CLEAN algorithm to address low signal to noise (SNR) targets. The Reformulated CLEAN Detector is presented which is shown to allow the detection of low SNR targets in the presence of large targets. Performance results are presented that show how good performance can be attained by combining the correlator, CLEAN Deconvolver and Reformulated CLEAN Detector					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT Same as Report (SAR)	18. NUMBER OF PAGES 6	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

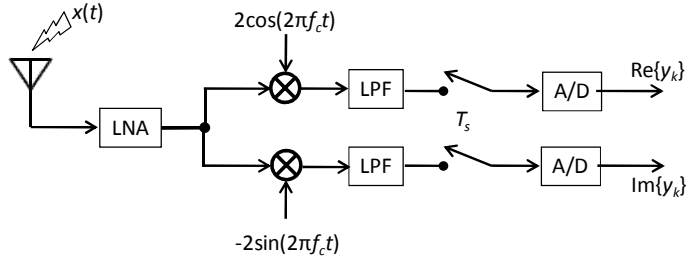
Some authors call this the least squares estimator [11] which it is. However, it is *important* to note that this is an estimator that conforms to the linear model and is therefore based on theorem 4.1 of [12, page 85-86], the Minimum Variance Unbiased (MVU) Estimator for \mathbf{c} . This means, for example, that (2) will give a smaller mean square error in estimating \mathbf{c} than the matched filter even though the matched filter gives the maximum SNR for same problem. (This is verified in [9].) The advantage of using (2) as a starting point for CLEAN is that its performance is optimal, it is easy to calculate and gives a lower bound for the estimation error.

II. SIGNAL MODEL

The problem that this paper addresses is detecting small targets in the presence of a large target under the condition of low SNR. In order to remove much of the artificiality of a baseband only analysis this paper uses a simulation of a modulated carrier pulse. This gives a signal model as

$$x(t) = s(t) \cos(2\pi f_c t) * \sum_{i=1}^q \alpha_i \delta(t - t_i) + n(t) \quad (3)$$

where $*$ is convolution, $s(t)$ is the baseband pulse, f_c is the carrier frequency, q is the number of targets (or scatterers), α_i is the complex amplitude of the target, $\delta(\cdot)$ is the Dirac delta function, t_i is the range induced time delay and $n(t)$ is white Gaussian noise. Figure 1 shows the standard receiver processor that produces the data vector \mathbf{y} based on the returned signal $x(t)$.



Equivalent Processing Block Diagram

Figure 1.

In order to do a digital simulation of Figure 1, a Matlab simulation was coded that implemented the processing in Figure 2. In this simulation the initial sample rate was set at five times ($M=5$) the ultimate sample rate required for the Nyquist sampling of $s(t)$. This allowed for setting target locations at fractional sample intervals.

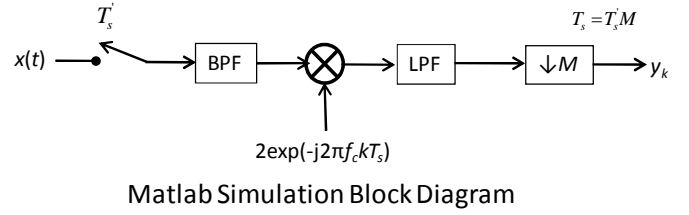


Figure 2

Now it is instructive to relate Figure 2 and (3) to (1) and (2). The PSF is \mathbf{w} and is equal to the output \mathbf{y} of Figure 2 when the input $x(t)$ is simply $s(t)\cos(2\pi f_c t)$, \mathbf{n} is equal to the output \mathbf{y} when $x(t) = n(t)$, and the elements of \mathbf{c} are $c_k = \alpha_k$ where $k = t/T_s$ rounded to the nearest integer.

III. REFORMULATED CLEAN DETECTOR

While the CLEAN Deconvolver is the preferred processor to detect and estimate closely spaced targets, it can have significant performance problems in situations of low SNR. In this case the preferred processor is the matched filter, however the range-time sidelobes can interfere with the detection of smaller targets. The approach to overcome these problems is to develop a new detector structure that combines the benefits of the CLEAN Deconvolver and the matched filter.

The original CLEAN Detector was shown to be equivalent to a deconvolver [9]. It also handled the situation of eclipsed targets. This new version of the CLEAN Detector will assume that there are no eclipsed targets (as the CLEAN Deconvolver) and will simultaneously act as a matched filter for small targets while eliminating large targets.

To begin with it is assumed that all large targets (i.e., targets large enough to suppress small targets with their sidelobes) have been detected. This can be done either by using the CLEAN Deconvolver or the matched filter. The amplitude and position of these targets are used to estimate \mathbf{c}^+ which will contain the amplitude of large targets and zero for all other elements. To derive this new detector the signal part of \mathbf{y} for a target in the k 'th range cell model is defined as

$$\mathbf{y}_s = \tilde{\mathbf{W}} \delta_k \alpha_k \quad (4)$$

where δ_k is all zeros except for the k 'th element. The interference part of \mathbf{y} (large targets and noise) is given by

$$\mathbf{y}_I = \tilde{\mathbf{W}}^H \mathbf{c}^+ + \mathbf{n} \quad (5)$$

Based on (5) the correlation matrix of the interference is given as

$$\mathbf{R}_I = E\{(\tilde{\mathbf{W}}^H \mathbf{c}^+ + \mathbf{n})(\tilde{\mathbf{W}}^H \mathbf{c}^+ + \mathbf{n})^H\} = \tilde{\mathbf{W}}^H \mathbf{c}^+ \mathbf{c}^{+H} \tilde{\mathbf{W}} + \sigma_n^2 \mathbf{I} \quad (6)$$

where σ_n^2 is the noise power and \mathbf{I} is the identity matrix and the noise is assumed to be zero mean white Gaussian noise (WGN). Invoking the assumption that the large targets are independent complex random variables with variance defined as $|\mathbf{c}_k^+|^2$, the processor based on the optimum detector can be written as (see [13] for a more complete method of derivation)

$$\bar{\mathbf{c}} = \tilde{\mathbf{W}}(\tilde{\mathbf{W}}^H \mathbf{c}^+ \mathbf{c}^{+H} \tilde{\mathbf{W}} + \sigma_n^2 \mathbf{I})^{-1} \mathbf{y}. \quad (7)$$

This processor acts as a matched filter while canceling the previously detected large targets.

IV. STRATEGIES IN APPLYING CLEAN ALGORITHMS

Having multiple processors (i.e. correlator, CLEAN Deconvolver and Reformulated CLEAN Detector) allows the development of strategies to use all these processors to obtain the advantages of each. The first algorithm to try is the CLEAN Deconvolver. If the SNR is high enough, then the CLEAN Deconvolver will provide the best detection and amplitude estimation of the multiple targets in the radar's range gate. If the smallest targets of interest have a low SNR such that they can not be seen with the CLEAN Deconvolver, then the larger targets detected by the CLEAN Deconvolver can be applied to the Reformulated CLEAN Detector to allow it to detect the smallest targets. If the largest targets have such a small SNR that they can not be detected by the CLEAN Deconvolver, then they can be detected with the correlator. These targets can then be applied to the Reformulated CLEAN Detector, allowing the detection of the smaller targets. These strategies are illustrated in the next section.

V. EXAMPLE PERFORMANCE RESULTS

The example problem considered is that of detecting small targets in the presence of a single large target. For this simulation the target Doppler was ignored (set to zero). This was done for simplicity of presentation. The techniques of [10] have been found to be adequate for addressing the Doppler issue for this problem.

Challenges addressed in this simulation include fractional spacing of targets, imperfect knowledge of the PSF (i.e., corrupted by noise), low SNR coupled with a high time-bandwidth pulse. These issues taken together severely limit the performance of the CLEAN Deconvolver. The SNRs quoted are taken prior to pulse compression (i.e., SNR of a single sample).

The problem to be solved in this example is illustrated in Figure 3. Figure 3 shows the correlator output of a hypothetical radar using a 32 chip derivative phase shift keying waveform with each chip repeated 16 times (i.e., 512 chip waveform). The data $y(t)$ is sampled twice per chip to achieve the Nyquist rate. Thus, the transmitted waveform is 1024 samples and the response of the correlator is 2047 samples. Note that for the waveform used the range time sidelobes are only 17 dB down from the peak of the target

response. There is a large target of 35 dB SNR in the center and three smaller ones with a SNR of 26 dB. Only the large target can be reliably detected due to the range-time sidelobes of the waveform used. The problem to be solved is to detect the three small targets in the presence of the large target.

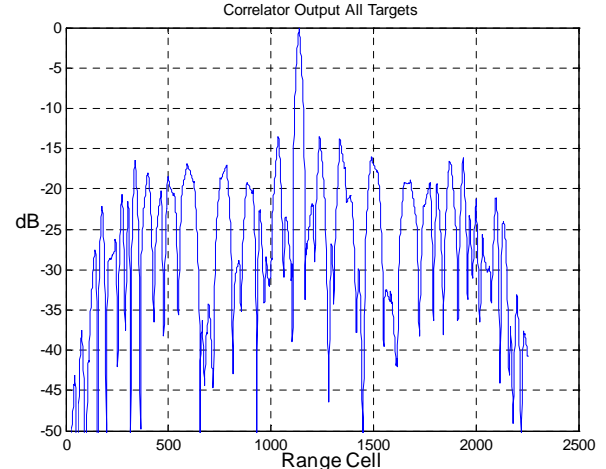


Figure 3. Four targets are in the complex but only one can be reliably detected due to sidelobes.

Figure 4 shows the output of the CLEAN Deconvolver processing the same data. Here all four targets can be seen. Figure 4 shows the strength of the CLEAN Deconvolver, namely that targets can be localized to a single range sample. It should also be noted that the targets are not injected at the range cell boundaries, but are put in straddling range cells by oversampling the input data and then filtering and downsampling. Figure 4 then shows the location of all four targets to nearest range sample.

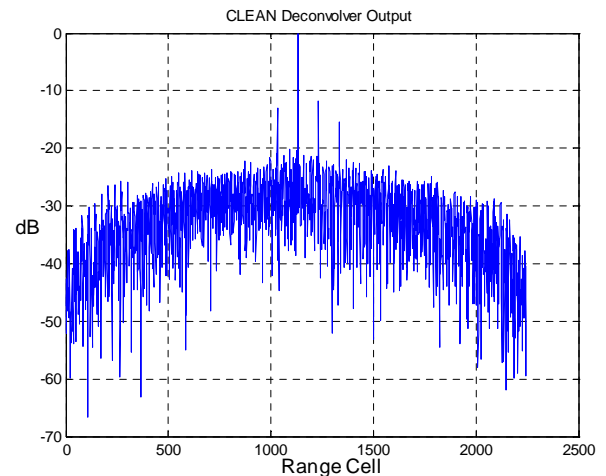


Figure 4. All Four Targets are seen by the CLEAN Deconvolver.

The next situation to be considered is the case where the smaller targets have such a low SNR (12 dB) that they can not be seen with the CLEAN Deconvolver. This is illustrated in Figure 5 where the noise and errors in the estimate of the PSF prevent the detection of the smaller targets. While not shown, the correlator will also fail to detect the smaller targets. However, for the correlator the reason for non-detection is the

range-time sidelobes of the waveform since the SNR of the small targets is large enough to be otherwise detected.

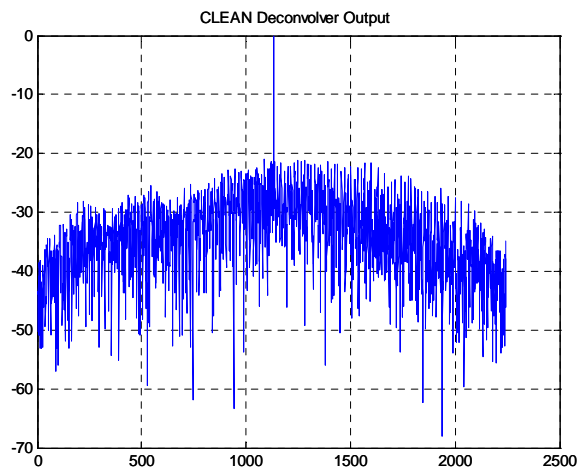


Figure 5. Only the largest target is seen by the CLEAN Deconvolver.

Using the CLEAN Deconvolver to estimate the previously detected target and putting it into \mathbf{c}^+ enables the Reformulated CLEAN Detector to be used to detect the smaller targets. The Reformulated CLEAN Detector has the effect of eliminating the large target that was detected by the CLEAN Deconvolver. This is illustrated in Figure 6. Here the three smaller targets are detected with virtually no interference from the large target. There is however, some minor raising of the time-sidelobes caused by the Reformulated CLEAN Detector.

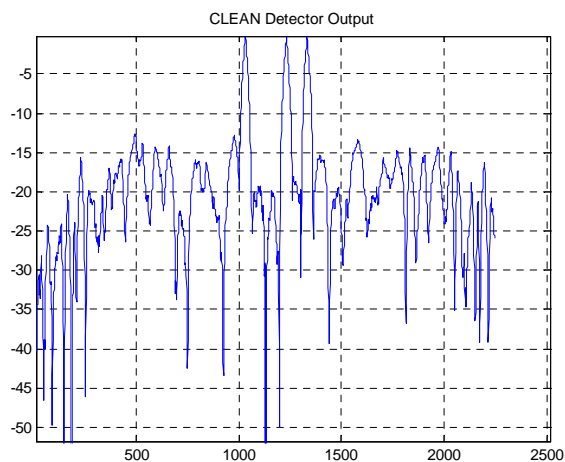


Figure 6. The Reformulated CLEAN Detector sees the three small targets while suppressing the large target.

The next situation to be analyzed is the case where the larger target has a SNR too low to be detectable by the CLEAN Deconvolver. (The next section quantifies the differences in output SNR of these processors.) For this example, the larger target SNR is 8 dB while the three smaller targets have a SNR of -9 dB. In Figure 7 we see the output of

the CLEAN Deconvolver where the large target is undetectable because the SNR is too low.

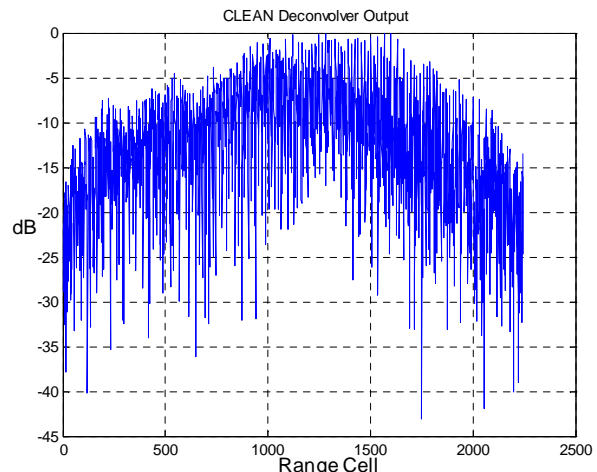


Figure 7. Output of the CLEAN Deconvolver. The SNR of the large target is too small to allow detection by the CLEAN Deconvolver.

For the waveform used here (time-bandwidth product of 512) it should increase the SNR by 27 dB. Hence the correlator can see the large target. This is shown in Figure 8 as well as the fact that the time-sidelobes from the correlator prevent the detection of the three smaller targets.

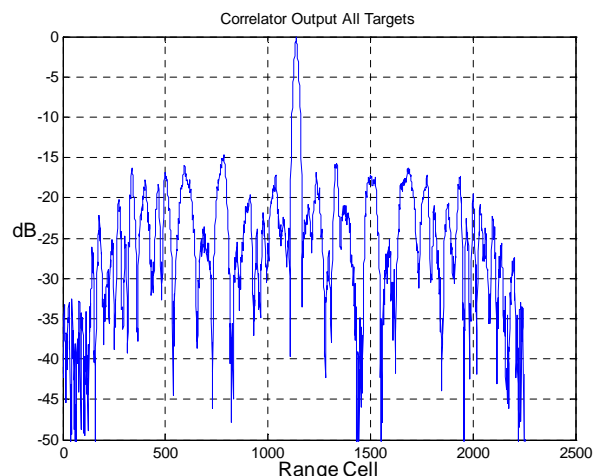


Figure 8. Output of the Correlator showing that the large target is detected.

Because the larger target can be detected by the correlator (see Figure 8), the amplitude and position of the target can be estimated by the correlator. This data is used to populate \mathbf{c}^+ indicating that the large target is now an interfering point scatter. Using this information, the Reformulated CLEAN Detector can detect the three smaller targets. This is demonstrated in Figure 9.

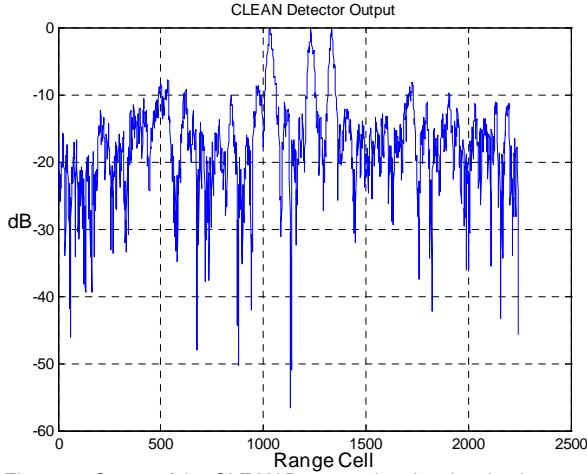


Figure 9. Output of the CLEAN Detector showing that the three small targets can be detected once the large target is removed.

While the examples in this section only had one large target, these techniques could be used for dealing with any number of large interfering targets, as long as they can be individually detected and removed. In fact these techniques work even better as the targets are larger having a large SNR.

VI. SNR COMPARISONS

Since it is well known that the matched filter produces the maximum output SNR for the situation of WGN this begs the question as to what is the SNR performance of the CLEAN Deconvolver and the Reformulated CLEAN Detector. In this section the SNR for CLEAN processors will be determined.

First for the CLEAN Deconvolver the signal output due to \mathbf{c} can be determined from (1) and (2) as

$$\hat{\mathbf{c}} = (\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H)^{-1} \tilde{\mathbf{W}}\tilde{\mathbf{W}}^H \mathbf{c} = \mathbf{c}. \quad (8)$$

Thus the signal output power is $|\mathbf{c}|^2$. The output noise correlation matrix using (2) and noting that \mathbf{n} is zero mean white Gaussian noise is

$$\mathbf{R}_n = E\{(\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H)^{-1} \tilde{\mathbf{W}}\mathbf{n}\mathbf{n}^H \tilde{\mathbf{W}}^H (\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H)^{-1}\} = \sigma_n^2 (\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H)^{-1}. \quad (9)$$

This gives the SNR for the k 'th range cell as

$$\text{SNR}_{\text{C_Decon}} = |\mathbf{c}_k|^2 / \sigma_n^2 (\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H)^{-1}_{(k,k)}. \quad (10)$$

Next the Reformulated CLEAN Detector output due to the signal is determined from (1) and (7) as

$$\hat{\mathbf{c}} = \tilde{\mathbf{W}}(\tilde{\mathbf{W}}^H \mathbf{c} + \mathbf{c}^H \tilde{\mathbf{W}} + \sigma_n^2 \mathbf{I})^{-1} \tilde{\mathbf{W}}^H \mathbf{c}. \quad (11)$$

The output noise correlation matrix for the Reformulated CLEAN Detector is determined as

$$\mathbf{R}_n = E\{\tilde{\mathbf{W}}(\tilde{\mathbf{W}}^H \mathbf{c} + \mathbf{c}^H \tilde{\mathbf{W}} + \sigma_n^2 \mathbf{I})^{-1} \mathbf{n}\mathbf{n}^H (\tilde{\mathbf{W}}^H \mathbf{c} + \mathbf{c}^H \tilde{\mathbf{W}} + \sigma_n^2 \mathbf{I})^{-1} \tilde{\mathbf{W}}^H\} \quad (12)$$

This can be simplified as

$$\mathbf{R}_n = \sigma_n^2 \tilde{\mathbf{W}}(\tilde{\mathbf{W}}^H \mathbf{c} + \mathbf{c}^H \tilde{\mathbf{W}} + \sigma_n^2 \mathbf{I})^{-1} (\tilde{\mathbf{W}}^H \mathbf{c} + \mathbf{c}^H \tilde{\mathbf{W}} + \sigma_n^2 \mathbf{I})^{-1} \tilde{\mathbf{W}}^H. \quad (13)$$

Based on this the SNR for the Reformulated CLEAN Detector for the k 'th range cell is

$$\text{SNR}_{\text{C_Det}} = |\hat{\mathbf{c}}_k| / \mathbf{R}_n(k,k) \quad (14)$$

where $\hat{\mathbf{c}}$ and \mathbf{R}_n are defined in (11) and (13) respectively.

The ratio of the SNR's for the two CLEAN processors to the matched filter is given in Figure 10. Figure 10 gives the relative performance for the example calculations in the previous section with one large target. Thus it can be seen that the Reformulated CLEAN Detector suffers almost no loss except in the vicinity of the large target that it is suppressing. In contrast the CLEAN Deconvolver suffers significant SNR loss compared to the matched filter. It should be noted that the loss is waveform dependent and is likely to be large for the large time-bandwidth product (i.e., 512) waveforms used in the previous section.

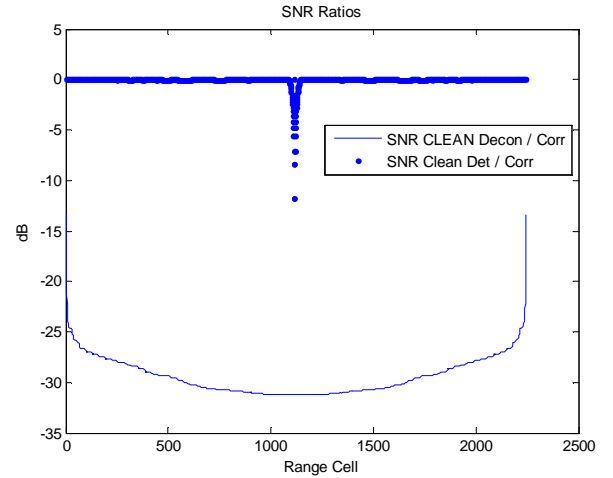


Figure 10. This figure shows the SNR disadvantage the CLEAN Deconvolver and the CLEAN Detector have compared with the Correlator

VII. COMPARISONS TO OTHER APPROACHES

At the suggestion of the reviewers this section compares and contrasts the approach in this paper to two others. The first is [8] which applies CLEAN to Space Time Adaptive Processing (STAP) problem. The authors apply the two dimensional CLEAN algorithm (much like radio astronomers [2]) to resolve targets and clutter. The authors use Taylor weighting to lower the sidelobes of the PSF to aid in the function of the CLEAN algorithm to resolve mainlobe and sidelobe responses. The use of this data is to remove array calibration errors. Thus, the measure of performance is not in detecting all the clutter or target sources but in how well the output of CLEAN allows the data cube to be calibrated such that subsequent processing can reliably detect targets. Probably the biggest difference in this approach and this paper (besides that [8] is a two dimensional problem) is that the sidelobes of the PSF are controlled to limit sidelobes. The PSFs in this paper had range-time sidelobes on the order of 17 dB and one of the strengths of this approach is to deal with PSFs with high range-time sidelobes.

The next paper to compare is [11] which uses a technique called Adaptive Pulse Compression (APC). The APC approach it to modify the correlator based on nearby targets to minimize the mean square error in estimating the range cell of interest. The statistical approach of [11] is Bayesian while that of this paper is a classical approach. Thus [11] assumes that \mathbf{c} is a random vector of known distribution and seeks to minimize the mean error in estimating it. In contrast the CLEAN Deconvolver assumes that \mathbf{c} is deterministic and produces an unbiased estimate of \mathbf{c} that has an error variance that is lower than all other estimators. The CLEAN Deconvolver does not have to estimate any intermediate values to produce its estimate of \mathbf{c} as is the case of APC. The Reformulated CLEAN Detector on the over hand is derived using detection theory as opposed to estimation theory. To apply it one must independently estimate \mathbf{c}^+ (vector of large targets). The Reformulated CLEAN Detector can also be described as a whitening matched filter. In addition, its optimality claim is that it produces a test statistic that maximizes the probability of detection for a given false alarm probability as long as \mathbf{n} and \mathbf{c}^+ are jointly Gaussian distributed.

VIII. SUMMARY AND CONCLUSION

This paper extended the previously developed CLEAN Deconvolver and CLEAN Detector by developing the Reformulated CLEAN Detector. Based on these processors and the correlator, strategies were described as to how these processors could be combined to address the problem of low SNR targets. The potential performance of this approach was then given in section V where it was shown that small targets could be detected in the presence of large targets.

REFERENCES

- [1] J. A. Hogbom, Aperture synthesis with non-regular distribution of interferometer baselines," *Astron. Astrophys. Suppl.*, vol. 15, 1974, pp.417–426.
- [2] Ronny Levanda and Amir Leshem, "Synthetic aperture radio telescopes," *IEEE Signal Processing Magazine*, Vol 27 No. 1 Jan. 2010, pp. 14-29.
- [3] J.Tsao and B. D. Steinberg, "Reduction of sidelobe and speckle artifacts in microwave imaging: The CLEAN technique," *IEEE Transactions on Antennas and Propagation*, vol. 36, Apr. 1988.
- [4] I.-S. Choi and H.-T. Kim, "One-dimensional evolutionary programming-based CLEAN", *Electronics Letters*, Vol. 37, No. 6, 15 March 2001.
- [5] R. Bose, A. Freedman, and B. D. Steinberg, "Sequence CLEAN: A modified deconvolution technique for microwave images of contiguous targets," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 38, no. 1, Jan. 2002. pp. 89-97.
- [6] R. Bose, "sequence CLEAN technique using breeder genetic algorithm for contiguous RADAR target images with high sidelobes," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, no. 1, Jan. 2003. pp. 368-373.
- [7] H. Deng, "Effective CLEAN algorithms for performance-enhanced detection of binary coding radar signals", *IEEE Transactions on Signal Processing*, Vol. 52, No. 1, January 2004.
- [8] G. R. Legters and J. R. Guerci, "Physics-based airborne GMTI radar signal processing," in *Radar Conference*, 2004. Proceedings of the IEEE, 2004, pp. 283-288.
- [9] T. L. Foreman, "Reinterpreting the CLEAN Algorithm as an optimum detector," Presented at the 2006 National IEEE Radar Conference, Verona, NY, April 24-27 , 2006.
- [10] Terry L. Foreman, "Adapting the CLEAN Deconvolver and CLEAN Detector to Doppler uncertainty," *IEEE Radar Conference*, 2007
- [11] Shannon K. Blunt, Karl Gerlach, "Adaptive Pulse Compression via MMSE estimation," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 42, no. 2, April 2006, pp 572-584.
- [12] Steven M. Kay, *Fundamentals of Statistical Signal Processing Estimation Theory*, Englewood Cliffs: Prentice Hall, 1993.
- [13] T. L. Foreman and S.G. Wilson, "Detection in distributed Rayleigh clutter," Presented at the IEEE 1998 National Radar Conference, Dallas, TX, 12-13 May 1998.